Problem Session 3

EE3713
Chelberg

Problem 1

Let X be a random variable that with PMF

\[
P_X(x) = \begin{cases} 
\frac{x^2}{a} & \text{if } x = -3, -2, -1, 0, 1, 2, 3 \\
0 & \text{Otherwise}
\end{cases}
\]

a). Find a and \( E[X] \).
b). What is the PMF of the random variable \( Z = (X - E[X])^2 \)
c). Using part b, compute the variance of \( X \).
d). Compute the variance of \( X \) using the formula \( Var(X) = \sum (x - E[X])^2 P_X(x) \)
e). Compute the variance of \( X \) using the formula \( Var(X) = E[X^2] - (E[X])^2 \)

Problem 2

Each of \( n \) files is saved independently onto either hard drive A (with probability \( p \)) or hard drive B (with probability \( 1 - p \)). Let \( A \) be the total number of files selected for drive A and let \( B \) be the total number of files selected for drive B.

(a) Determine the PMF, expected value, and variance for random variable \( A \).

(b) Evaluate the probability that the first file to be saved ends up being the only one on its drive.

(c) Evaluate the probability that at least one drive ends up with a total of exactly one file.

(d) Evaluate the expectation and the variance for the difference, \( D = A - B \).

Problem 5

Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
Problem 6

A company has 10 software engineers and 6 electrical engineers. In how many ways can a committee of 4 engineers be formed from them such that the committee must contain exactly 1 electrical engineer?

Problem 7

From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. In how many ways can the committee be formed if one man and one woman refuses to serve together?