HW Assignment 2 (Due date: September 21, Monday)

6. [Asymptotic Notation, 10 points] Prove or disprove:
   a) $3^{n+1} = O(3^n)$
   b) $2^{2n} = O(2^n)$
   c) $3n^2 \lg n + 4n = O(n^3)$
   d) $3n^2 \lg n + 4n = O(n^2 \lg n)$
   e) $3n^2 \lg n + 4n = O(n^2 \sqrt{n})$
7. [Asymptotic Notation, 5 points] Prove $\lg(n!) = \theta(n \lg n)$.
   [Hint: use one of Stirling’s approximations (3.18 or 3.20 on page 57)].
8. [Substitution Method, 15 points] Show that the solution to:
   \[
   T(6) = 1 \\
   T(n) = 3T(\lfloor n/3 \rfloor + 4) + n, \text{ for } n > 6 \\
   \]
   is $O(n \lg n)$.
9. [Master Method, 10 points] Use the master method to give tight asymptotic bounds for the following recurrences:
   a) $T(n) = 4T(n/2) + n$.
   b) $T(n) = 4T(n/2) + n^2$.
   c) $T(n) = 4T(n/2) + n^3$.
   d) $T(n) = 2T(n/4) + \sqrt{n}$.
   e) $T(n) = 2T(n/4) + n^2$.
10. [Master Method, 5 points] The recurrence $T(n) = 10T(n/3) + n^2$ describes the running time of an algorithm $A$. A competing algorithm $A'$ has a running time of $T'(n) = aT'(n/9) + n^2$. What is the largest integer value for $a$ such that $A'$ is asymptotically faster than $A$?
11. [Recurrence, 20 points] Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.
a) \( T(n) = 2T(n/2) + n^3. \)
b) \( T(n) = T(9n/10) + n. \)
c) \( T(n) = 16T(n/4) + n^2. \)
d) \( T(n) = 7T(n/3) + n^2. \)
e) \( T(n) = 7T(n/2) + n^2. \)
f) \( T(n) = 2T(n/4) + \sqrt{n}. \)
g) \( T(n) = T(n - 1) + n. \)
h) \( T(n) = T(\sqrt{n}) + 1. \)

12. **[Recurrence, 5 points (*)]** Solve the recurrence \( T(n) = 2T(\sqrt{n}) + 1 \) by making a change of variable. The solution should be asymptotically tight (i.e. use the \( \Theta \) notation). Do not worry about whether values are integral.

13. **[Design & Analysis, 10 points (*)]** Exercise 2.3-7, page 39.