HW Assignment 2 (Due date: September 22, Monday)


6. [Asymptotic Notation, 10 points] Prove or disprove:
   a) $3^{n+1} = O(3^n)$
   b) $2^{2n} = O(2^n)$
   c) $3n^2 \lg n + 4n = O(n^3)$
   d) $3n^2 \lg n + 4n = O(n^2 \lg n)$
   e) $3n^2 \lg n + 4n = O(n^2 \sqrt{n})$

7. [Asymptotic Notation, 5 points] Prove $\lg(n!) = \theta(n \lg n)$.
   [Hint: use one of Stirling’s approximations (3.18 or 3.20 on page 57)].

8. [Substitution Method, 15 points] Show that the solution to:
   \[
   T(6) = 1 \\
   T(n) = 3T(\lfloor n/3 \rfloor + 4) + n, \text{ for } n > 6
   \]
   is $O(n \lg n)$.

9. [Master Method, 10 points] Use the master method to give tight asymptotic bounds for the following recurrences:
   a) $T(n) = 4T(n/2) + n$
   b) $T(n) = 4T(n/2) + n^2$
   c) $T(n) = 4T(n/2) + n^3$
   d) $T(n) = 2T(n/4) + \sqrt{n}$
   e) $T(n) = 2T(n/4) + n^2$

10. [Master Method, 5 points] The recurrence $T(n) = 10T(n/3) + n^2$ describes the running time of an algorithm $A$. A competing algorithm $A'$ has a running time of $T'(n) = aT'(n/9) + n^2$. What is the largest integer value for $a$ such that $A'$ is asymptotically faster than $A$?

11. [Recurrence, 20 points] Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.
12. [**Recurrence, 5 points (**)]: Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ by making a change of variable. The solution should be asymptotically tight (i.e. use the $\Theta$ notation). Do not worry about whether values are integral.

13. [**Design & Analysis, 10 points (**)]: Exercise 2.3-7, page 39.

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a) $T(n) = 2T(n/2) + n^3$.  
b) $T(n) = T(9n/10) + n$.  
c) $T(n) = 16T(n/4) + n^2$.  
d) $T(n) = 7T(n/3) + n^2$.  
e) $T(n) = 7T(n/2) + n^2$.  
f) $T(n) = 2T(n/4) + \sqrt{n}$.  
g) $T(n) = T(n-1) + n$.  
h) $T(n) = T(\sqrt{n}) + 1$. 

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