ABSTRACT

Motion estimation is a very important method for improving image quality by compensating the cardiac motion at the best phase reconstructed. We tackle the cardiac motion estimation problem using an image registration approach. We compare the performance of three gradient-based registration methods on clinical data. In addition to simple gradient descent, we test the Nesterov accelerated descent and conjugate gradient algorithms. The results show that accelerated gradient methods provide significant speedup over conventional gradient descent with no loss of image quality.

1. INTRODUCTION

Cardiac computed tomography (or cardiac CT) imaging technology has been developed to provide clear, detailed images of cardiac structures, in particular the coronary artery trees at the different cardiac phases. The image quality of cardiac CT is largely determined by the temporal resolution of CT systems, which is limited by the fastest possible gantry rotation speed. With the increased heart beat rate during the data acquisition, the degradation of image quality caused by motion artifacts is an inevitable and serious problem even in high-end CT systems. Motion compensation method, which is based on incorporating the cardiac motion model into a reconstruction algorithm, is believed to use all the acquired data and have the potential to provide artifact-free image with reduced x-ray dose.

As the fundamental part of the motion compensation method, motion estimation is a very important challenge in CT-based cardiac imaging due to the intrinsic complexity of cardiac motion. In this paper, we use image registration for estimating cardiac motion, and then use the latter for performing motion-compensated reconstruction. See for a similar approach. Instead of reconstructing the entire cardiac cycle as in, we concentrate only on improving image quality at a quiet cardiac phase. The main focus of this paper is to explore fast algorithms for image registration, which is the key step in our reconstruction algorithm. We will discuss three gradient-based optimization approaches to image registration, and then compare them based on speed and reconstruction quality.

2. ESTIMATING LOCAL MOTION THROUGH IMAGE REGISTRATION

The main idea of our approach is as follows. Let $Q$ denote the fraction corresponding to the quiet cardiac phase (e.g., $Q = 70\%$). Let $N$ denote a small fraction (say, $N = 5\%$). Our goal is to reconstruct a motion-compensated image at phase $Q$. The algorithm performs the following steps. (1) Reconstruct three images $f_k$, $k = 1, 2, 3$, at phases $Q - N$, $Q$, and $Q + N$, respectively. At this stage motion is assumed to be zero. (2) Register $f_1$ to $f_2$, and $f_3$ to $f_2$. (3) Assuming that motion of every pixel is linear in time, create a motion model based on the results of registration. (4) Reconstruct the final image at phase $Q$ using the motion models determined at step (3). To improve efficiency we use the idea of partial angle reconstructions (also known as subphasic volumes) suggested in. The volumes that are reconstructed using views to the left of $Q$ are warped using the motion model obtained by registering images $f_1$ and $f_2$. The volumes that are reconstructed using views to the right of $Q$ are warped using the motion model obtained by registering images $f_3$ and $f_2$.

Further author information: Send correspondence to A. Zamyatin at AZamyatin@tmriusa.com.

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3. IMAGE REGISTRATION METHODS

3.1 An optimization framework for image registration

Let $f$ and $g$ be the two volumes to be registered (known on a fine grid $x_1, \ldots, x_N$). We seek a deformation which will transform $g$ into $f$. Due to the elasticity of cardiac deformations, the deformation we seek must be nonrigid. One way of describing a general deformation is to specify a direction of motion on a possibly coarser grid of points in the image $g$. If $\tilde{x}_1, \ldots, \tilde{x}_N$ represent the points on this motion grid, we will denote by $\Psi(\tilde{x}_k)$ the motion at each point. Moreover, this motion can be trilinearly interpolated onto the finer image grid $x_1, \ldots, x_N$. Hence, we seek a $\Psi$ for which $f(x_k) \approx g(x_k + \Psi(x_k))$ for points $x_k$ in the image grid. Note that since the images $f, g$ are only known on the image grid $x_1, \ldots, x_N$, the value $g(x_k + \Psi(x_k))$ must also be calculated by trilinear interpolation of $g$. Finally, we also have the restriction that the deformation $\Psi$ needs to be sufficiently smooth; indeed cardiac motion is a physical process and thus is at least continuous. Thus for accurate motion estimation it is desirable to control the regularity of $\Psi$. We can meet all of these requirements by finding our motion function as the solution of the following optimization problem:

$$
\min_{\Psi} \left( \sum_{i=1}^{N} |f(x_i) - g(x_i + \Psi(x_i))|^2 + \lambda \sum_{j=1}^{n} |\nabla \Psi(x_j)|^2 \right),
$$

The first term in the objective function is a measure of the misfit of the two volumes for a given motion $\Psi$, whereas the second term is a regularization term to encourage the smoothness of the deformation $\Psi$. Also, $\lambda$ is a regularization parameter which controls the tradeoff between smoothness of motion and accuracy of registration.

3.2 Gradient-based methods for image registration

Given our application, it is important for our registration algorithm to run quickly. Thus we turn to gradient-based methods, many of which are relatively computationally inexpensive. Let $F(x)$ denote our objective function, where $x$ is a vector of the values of $\Psi$ on the motion grid and $F$ is the of the form (1). Our baseline optimization method is simple gradient descent (Algorithm 1).

**Algorithm 1 Simple Gradient Descent**

1: Choose $x_0 \in \mathbb{R}^n$, and choose a tolerance $\epsilon$.
2: for $k = 0, 1, \ldots, \text{do}$
3: (Gradient evaluation) Evaluate $\nabla F(x_k)$. This is the direction of steepest ascent.
4: (Line search) Find $\alpha_k = \arg \min_{\alpha} F(x_k - \alpha \nabla F(x_k))$. This step finds the best objective function value along the gradient.
5: (Update $x$) Set $x_{k+1} = x_k - \alpha_k \nabla F(x_k)$.
6: end for
7: Stop the algorithm when either $x_k$ or $F(x_k)$ changes by less than $\epsilon$.

One improvement to gradient descent was proposed by Nesterov. He proposed to add a momentum term $(x_k - x_{k-1})$ at each iteration. That is, we evaluate the gradient not at the previous iterate $x_k$, but at a point $y_k = x_k + \beta_k(x_k - x_{k-1})$. This represents a shift from $x_k$ by $\beta_k(x_k - x_{k-1})$. The latter quantity is the momentum term, which is proportional to the direction the algorithm took in the previous iteration. This leads to the Nesterov accelerated gradient descent (Algorithm 2).

A third gradient-based method is the nonlinear conjugate gradient (CG) method. This is an extension of the conjugate gradient method for solving large symmetric linear systems $Ax = b$. Instead of following the gradient at each iteration, the CG method proceeds along a conjugate gradient direction $s_k$. This is still a gradient-based method because $s_k$ is computed from $\nabla F(x_k)$. The CG method is detailed in Algorithm 3.
3.3 Multiresolution convergence acceleration scheme

All the gradient-based methods described above can be incorporated in a multiresolution scheme to accelerate the registration and avoid local minima. The basic idea of the multiresolution scheme is to divide the whole registration process into several levels. Begin by representing the registration and avoid local minima. The basic idea of the multiresolution scheme is to divide the whole

Algorithm 2 Nesterov Accelerated Gradient Descent

1: Choose $x_0 \in \mathbb{R}^n$, set $y_0 = x_0$ and choose a tolerance $\epsilon$.
2: Choose a sequence $\beta_0, \beta_1, \ldots$ in the interval $[0, 1]$.
3: for $k = 1, 2, \ldots$ do
4: (Gradient evaluation) Evaluate $\nabla F(y_{k-1})$.
5: (Line search) Find $\alpha_{k-1} = \arg \min_{\alpha} F(y_{k-1} - \alpha \nabla F(y_{k-1}))$.
6: (Update $x$) Set $x_k = y_{k-1} - \alpha_{k-1} \nabla F(y_{k-1})$.
7: (Update $y$) $y_k = x_k + \beta_k (x_k - x_{k-1})$.
8: end for
9: Stop the algorithm when either $x_k$ or $F(x_k)$ changes by less than $\epsilon$.

Algorithm 3 Nonlinear Conjugate Gradient

1: Choose $x_0 \in \mathbb{R}^n$, set $s_0 = \nabla F(x_0)$ and choose a tolerance $\epsilon$.
2: for $k = 1, 2, \ldots$ do
3: (Gradient evaluation) Evaluate $\nabla F(x_k)$.
4: (Compute $\beta_k$) Find $\beta_k$ from a formula including only $\nabla F(x_k)$ and $\nabla F(x_{k-1})$. A popular choice is the Polak-Ribiére formula.\textsuperscript{6}
5: (Update the conjugate direction) Let $s_k = \nabla F(x_k) + \beta_k s_{k-1}$.
6: (Line Search) Find $\alpha_k = \arg \min_{\alpha} F(x_k - \alpha s_k)$.
7: (Update $x$) $x_{k+1} = x_k - \alpha_k s_k$.
8: end for
9: Stop the algorithm when either $x_k$ or $F(x_k)$ changes by less than $\epsilon$.

4. NUMERICAL RESULTS

We implemented the three image registration methods from the previous section on a GPU and compared their runtimes on the same pairs of images $f$ and $g$. The experimental data we used is a clinical Cardiac CT image with the patient heart rate at 67.6 bpm. The quiet cardiac phase is selected at 70% ($Q = 70\%$), with two other Phases at 56.9% ($Q - N$) and 83.1% ($Q + N$).

The per-iteration complexities of the three algorithms are approximately the same, so their total runtimes are proportional to their numbers of iterations. The results of our numerical experiments show that simple gradient descent is the slowest of the three methods, that Nesterov accelerated gradient descent improves this performance, and that CG is the fastest of the three methods. See Fig. 2 for the convergence graphs of the three methods.

Fig. 3 shows the results of the motion artifacts reduction after motion compensation at Phase 70%. To quantitatively evaluate the image quality of our motion compensated results, we compared several known metrics before and after motion compensation at two regions of interest: Right Coronary Arteries (RCA) and Left Coronary Arteries (LCA). These metrics include standard deviation (STD), Positivity, Entropy, and Edge Entropy.\textsuperscript{2, 7, 8}

The values of the metrics for each method are shown in Table 1.
Figure 1. Multiresolution registration with 3 levels.

Figure 2. Comparision of optimization convergence for Simple Gradient Descent, Nesterov Accelerated Gradient, and CG.
(a) Convergence graph for all three resolution levels from Phase 56.9% to Target Phase 70.0%; (b) Convergence graph for the coarse level only from Phase 56.9% to Target Phase 70.0%; (c) Convergence graph for all three resolution levels from Phase 83.1% to Target Phase 70.0%; (d) Convergence graph for the coarse level only from Phase 83.1% to Target Phase 70.0%.
Figure 3. Motion artifacts reduction: (a) Target Phase 70.0% before motion compensation (red circles show the area of artifacts around coronary arteries); (b) Motion compensated results by using Simple Gradient Descent, (c) Nesterov Accelerated Gradient, and (d) CG.

<table>
<thead>
<tr>
<th>Group</th>
<th>STD</th>
<th>Positivity</th>
<th>Entropy</th>
<th>Edge Entropy</th>
</tr>
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<tbody>
<tr>
<td>Before MC (RCA)</td>
<td>126.6</td>
<td>2.202 × 10^3</td>
<td>6.219</td>
<td>65.1%</td>
</tr>
<tr>
<td>Simple Gradient Descent (RCA)</td>
<td>124.8</td>
<td>2.133 × 10^3</td>
<td>6.196</td>
<td>61.1%</td>
</tr>
<tr>
<td>Nesterov Accelerated Gradient (RCA)</td>
<td>124.9</td>
<td>2.137 × 10^3</td>
<td>6.197</td>
<td>61.2%</td>
</tr>
<tr>
<td>CG (RCA)</td>
<td>124.1</td>
<td>2.142 × 10^3</td>
<td>6.193</td>
<td>61.8%</td>
</tr>
<tr>
<td>Before MC (LCA)</td>
<td>137.6</td>
<td>4.185 × 10^3</td>
<td>6.249</td>
<td>61.1%</td>
</tr>
<tr>
<td>Simple Gradient Descent (LCA)</td>
<td>130.0</td>
<td>3.207 × 10^3</td>
<td>6.154</td>
<td>51.6%</td>
</tr>
<tr>
<td>Nesterov Accelerated Gradient (LCA)</td>
<td>129.9</td>
<td>3.197 × 10^3</td>
<td>6.154</td>
<td>52.5%</td>
</tr>
<tr>
<td>CG (LCA)</td>
<td>131.3</td>
<td>3.306 × 10^3</td>
<td>6.160</td>
<td>52.4%</td>
</tr>
</tbody>
</table>

Table 1. Quantitative metrics of motion artifacts around Right Coronary Arteries (RCA) and Left Coronary Arteries (LCA). Note that smaller values in each column correspond to higher image quality.
5. CONCLUSION

The proposed motion compensation approach results in the significant reduction of motion artifacts. All three methods provide nearly identical image quality, but the two accelerated descent methods are three to four times faster than simple gradient descent. The CG method is about two times faster even than the Nesterov method. However, we noticed that in some experiments CG cannot quite reach the same objective function value as Nesterov. This results in a minor loss of image quality – see the bottom panel in Figure 3d and the bottom row in Table 1. In future work, we plan to verify our motion estimation results on additional clinical datasets.

REFERENCES


